Joint Inventory and Distribution Strategy for Online Sales with a Flexible Delivery Option

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Outline

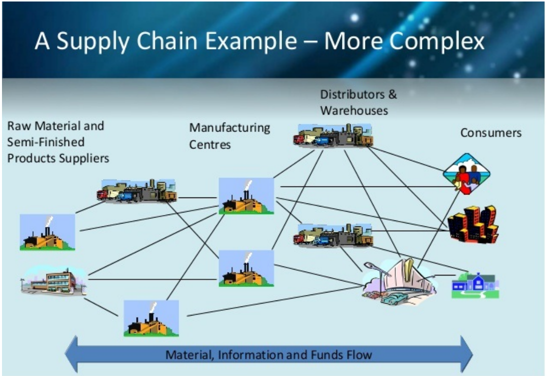
Section 1

Introduction

Supply Chain Management

Definition SCM

Supply Chain Management (SCM) is controlling and planning of all supply chain activities.



Figure

Definition IM

Inventory Management (IM) is the controlling of products kept in stock over time.

- The main purpose of IM is to answer;
  "How much to order?" and "When to order?"
- Traditional - Find inventory policy for a given demand pattern
- Knowing the demand pattern is important. Controlling and manipulating it?

Price Discounts in IM

- Quantity discounts, discounts for perishable products which has closer due date, discounts for pre orders etc.
- In this paper, we offer discounts to convince the customers for accepting later deliveries.
- Our main objective is to develop an inventory policy for a continuous review inventory-distribution system and its utilization by price discount.

Amazon Example

- Amazon offers incentives to convince customers for later deliveries
- Saving on delivery cost
- We focus on savings on inventory costs and delivery cost

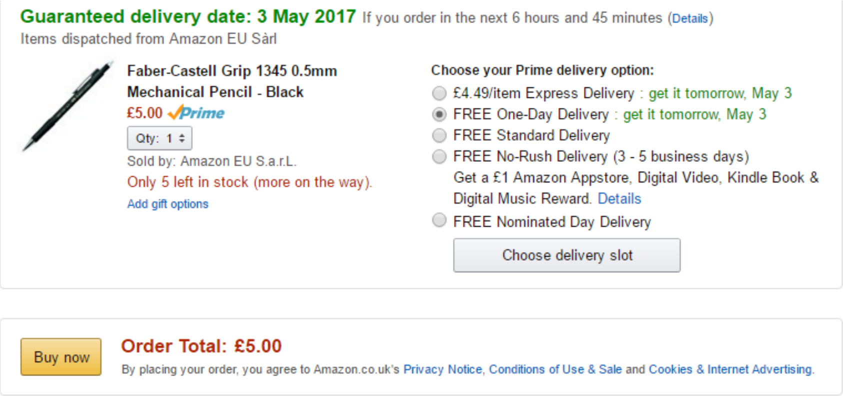


Figure: Example of Postponing Demand by Gift Card

Problem Definition

- Different customers have different willingness to wait for their orders.
- The time lags between ordering and delivery will vary with the sector. While books delivered within 1-5 days, in home improvement sector it could take weeks.
- Increasing customer lead time - preorder or postponement?
- Company can either outsource the delivery or do it by its own resources.

Problem Definition

- Constant demand rate with continuous review inventory system
- When inventory reaches a threshold, start offering discount
- Customer pays a deposit when they place the order and pays the remaining discounted price on delivery
- The postponed demands are satisfied by the next delivery from supplier
- Increasing the cycle time & postpone the order decision

Our Contributions

Joint inventory and distribution strategy for online sales with a flexible delivery option

- This paper contributes to the field of continuous review inventory models with NPV approach and postponed demand
- The fraction of time in which we postpone the demand is a decision that forms part of the inventory policy and the cash flow is dependent on the inventory policy.
- This situation makes AC not a nice approximation of NPV as present by Vander(2003).
- So far to our knowledge, this is the first study consider the postponed demand with NPV approach.
- Also we consider a distribution strategy which is dependent to the discount policy.

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Problem Description

Notation

Parameters

y	Constant demand rate per unit time $y > 0$
p	Sales price per unit of the product $p > w$
w	Cost price of a unit product
r	Discount amount per unit per delay, $\$/delay$, $0 \leq r < p - w$
g	Deposit paid by customers due to late delivery $\$$, $g < p - r$
S	Setup cost per replenishment, $\$/order$
$I(t)$	Inventory level at time t
U	Maximum waiting time promised to customers, *weeks*
Δt	The time between two consecutive delivery to customers
A	Size of the total distribution region, $miles^2$
ρ	Average distance from a customer in distribution region to the depot, *miles*
$n(k)$	Number of customers need to be served at the kth delivery
$d(k)$	Average distance travelled at the kth delivery, *miles*
$c(k)$	Cost of the delivery at time t due to distribution by company, $\$/delivery$
$c_u(k)$	Unit cost of the delivery by the company at the kth delivery, $\$/unit$
c_p	Delivery cost of outsourcing an item, $\$/unit$
β	Ratio of demand accepting the flexible delivery option, $0 < \beta < 1$
α	Opportunity cost of capital rate
ζ	Euclidean metric factor
γ	Vehicle operation cost per mile

Decision Variables

T	Inventory cycle time, $T > 0$
F	Fraction of time T that discount is not offered, $0 \leq F \leq 1$
$J(k)$	Binary variable to decide whether to outsource the delivery or not, $J(k) \in \{0, 1\}$

Problem Description

Pipeline & Payment Scheme

- We consider a single item, continuous review inventory and distribution system with constant demand rate and price discount.

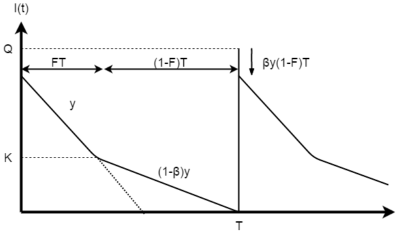

Figure: Inventory Flow

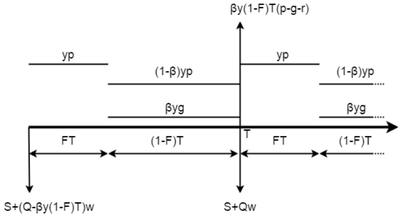

Figure: Cash Flow

- Distribution to customer locations is organized at discrete moments in time $(\Delta t, 2\Delta t, ..., k\Delta t, ...)$.
- Number of customers served at each delivery are as;

$$n(k) = \begin{cases} y\Delta t & \Delta t < k\Delta t \mod T \leq FT \\ y\Delta t - y\beta(k\Delta t \mod T - FT) & FT < k\Delta t \mod T < FT + \Delta t \\ (1-\beta)y\Delta t & FT + \Delta t \leq k\Delta t \mod T \leq T \\ n(1) + (1-F)Ty\beta & k\Delta t \mod T = \Delta t \text{ and } k > 1 \end{cases} \quad (1)$$

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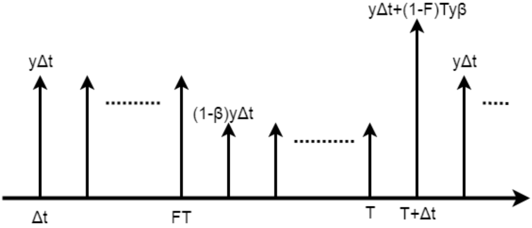

Figure: $n(k)$ values within an inventory cycle

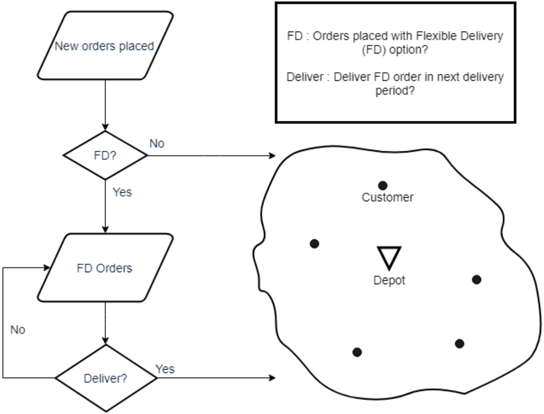

Figure: Process flow of order placement and delivery

Daganzo's Formula

The average distance travelled at the kth delivery is approximated in (2). This model is derived from the continuous approximation method developed in Daganzo and Newell 1985.

$$d(k) \approx 2\rho + \zeta(n(k))\sqrt{An(k)} \tag{2}$$

$$c_u(k) \approx \frac{\gamma d(k)}{n(k)} \tag{3}$$

where $\zeta(n(k)) \approx 0.57$ and γ is the vehicle operation cost per mile including driver's wage. If the warehouse is located inside the service region, Danganzo's formula can be simplified to $d(k) \approx \zeta\sqrt{A(n(k)+1)}$, where $\zeta \approx 0.71$.

Customer's Acceptance of Flexible Delivery

It is in line of expectation that more customers will be induced to accept the flexible delivery option the higher the discount and the lower the maximum waiting time and deposit value. We will examine the case that the fraction β is influenced by the discount amount r, maximum waiting time U, and deposit value g according to the following relationship:

$$\beta(r, U, g) = e^{-\frac{\theta(U, \frac{g}{p-r})}{r}}, \quad (4)$$

where $\theta(U, g/(p-r)$ is a factor that measures the *customer resistance* to the discount offer, which increases with U and $g/(p-r)$.

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Lemma

The objective function $ASTP(F, T)$ is a concave function under positive setup cost when the distribution cost is neglected.

Lemma

Let the T_{NPV} and T_{EOQ} are the optimal T values of NPV model with discount and classic EQO model, then the followings hold when the distribution cost is neglected;

$$T_{NPV} \geq T_{EOQ}$$

The discount amount to be offered is also an important parameter to decide. Although we assume it as parameter, we show the upper bounds for discount amount.

Lemma

The discount option is profitable when the maximum discount amount is as follows when the distribution cost is neglected;

$$r \leq \frac{\sqrt{\frac{8S\alpha w}{y}}}{2 - \sqrt{\frac{2S\alpha}{yw}}} \tag{5}$$

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Special Case 3

Distribution cost Only

Lemma

The number of demand at each delivery can be used as preliminary to decide on distribution as;

1. *Distribution is fully outsourced if;*

$$c_p \leq c_u^1 = \gamma\zeta\sqrt{\frac{A}{y\Delta t + Uy\beta}} \tag{6}$$

2. *Only the delivery of the demand with postponed demand can be considered and others are outsourced if;*

$$c_p \leq c_u^2 = \gamma\zeta\sqrt{\frac{A}{y\Delta t}} \tag{7}$$

3. *Whole distribution process is provided by the company if;*

$$c_p \geq c_u^3 = \gamma\zeta\sqrt{\frac{A}{(1-\beta)y\Delta t}} \tag{8}$$

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Solution Algorithm

Algorithm 1: Exhaustive Search Algorithm for NPV Model

return $T^*, F^*, maxprofit$
initialize; $maxprofit, T_{EOQ}, U$
for $k \geq \frac{T_{EOQ}}{\Delta t}$ **to** $\frac{1}{\Delta t}$ **do**
    for $j = 0$ **to** 100 **do**
        $T = k\Delta t$
        $F = 0.01j$
        if $(1 - F)T \leq U$ **then**
            Calculate $ASTP(F, T)$
            if $ASTP(F, T) \geq maxprofit$ **then**
                $T^* = T$
                $F^* = F$
                $maxprofit = ASTP(F, T)$
            end if
        end if
    end for
end for

Numerical Results

- In this section, we run some numerical experiments to show the impact of out postponement policy on both inventory and distribution policy.
- First, we assume that the distribution is fully outsourced and focus on the inventory profits.
- Next, we include the partial distribution by own delivery and show how the postponement affects the strategy.
- At the end, the savings on average travelled distance is presented by postponement policy.

Fully Outsourced Distribution

Financial Impact of Discount

Table: Impact of Discount, $p = 1.3w$, $w = 10, r = 0.01p$, $g = 0$

	EOQ		NPV (U=3 weeks)			NPV (U=6 weeks)			
Parameters	T	β	T	F	$\delta(\%)$	T	F	$\delta(\%)$	$r_{max}(\%p)$
$\alpha = 0.1$, $S = 50$	0.32	0.1	0.32	0.82	0.10	0.32	0.63	0.16	
		0.5	0.33	0.82	0.54	0.35	0.66	0.86	2.47
		0.9	0.35	0.83	0.99	0.40	0.70	1.63	
$\alpha = 0.1$, $S = 80$	0.40	0.1	0.41	0.86	0.13	0.40	0.70	0.23	
		0.5	0.42	0.86	0.67	0.44	0.73	1.16	3.13
		0.9	0.42	0.86	1.22	0.46	0.74	2.15	
$\alpha = 0.2$, $S = 50$	0.22	0.1	0.23	0.74	0.26	0.23	0.48	0.36	
		0.5	0.23	0.74	1.30	0.26	0.54	1.95	3.51
		0.9	0.26	0.77	2.37	0.29	0.59	3.74	
$\alpha = 0.2$, $S = 80$	0.28	0.1	0.28	0.79	0.31	0.29	0.59	0.50	
		0.5	0.30	0.80	1.60	0.31	0.62	2.57	4.47
		0.9	0.30	0.80	2.90	0.33	0.64	**4.80**	

Fully Outsourced Distribution

Effect of Financial Settings

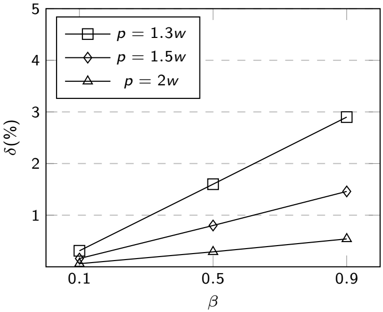

Figure: U=3 weeks

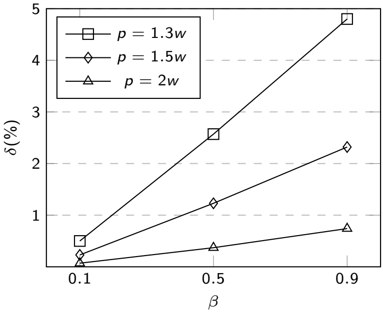

Figure: U=6 weeks

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Table: Impact of Deposit, $S = 80$, $\alpha = 0.2$, $p = 1.3w$, $w = 10, r = 0.01p$

Parameters	β	NPV (U=3 weeks) T	NPV (U=3 weeks) F	NPV (U=3 weeks) $\delta(\%)$	NPV (U=10 weeks) T	NPV (U=10 weeks) F	NPV (U=10 weeks) $\delta(\%)$
$g = 0$	0.1	0.28	0.79	0.31	0.29	0.40	0.56
	0.5	0.30	0.80	1.60	0.34	0.45	3.13
	0.9	0.30	0.80	2.90	0.37	0.49	6.15
$g = 0.5(p - r)$	0.1	0.28	0.79	0.34	0.29	0.35	0.88
	0.5	0.30	0.80	1.75	0.32	0.41	4.58
	0.9	0.30	0.80	3.19	0.35	0.46	8.52
$g = p - r$	0.1	0.28	0.79	0.38	0.29	0.35	1.20
	0.5	0.30	0.80	1.91	0.31	0.39	6.08
	0.9	0.30	0.80	3.47	0.31	0.39	11.03

Partial Outsourcing

Table: Impact of Distribution Cost, $S = 80, r = 0.01p$, $\beta = 0.5$, $g = p - r$

Unit Cost	Dist. Strategy	U=3 weeks			U=6 weeks		
		T	F	δ_{dist}	T	F	δ_{dist}
$c_p = c_u^1 = 0.05w$	Outsource	0.30	0.80	2.5	0.29	0.59	5.0
$c_p = c_u^1 = 0.10w$	Outsource	0.30	0.80	2.5	0.29	0.59	5.0
$c_p = c_u^2 = 0.05w$	Partial Outsource	0.26	0.77	8.3	0.26	0.54	17.8
$c_p = c_u^2 = 0.10w$	Partial Outsource	0.26	0.77	8.3	0.21	0.43	21.9
$c_p = c_u^3 = 0.05w$	Local delivery	0.30	0.80	3.3	0.29	0.59	7.6
$c_p = c_u^3 = 0.10w$	Local delivery	0.27	0.78	3.5	0.26	0.54	8.4

Partial Outsourcing

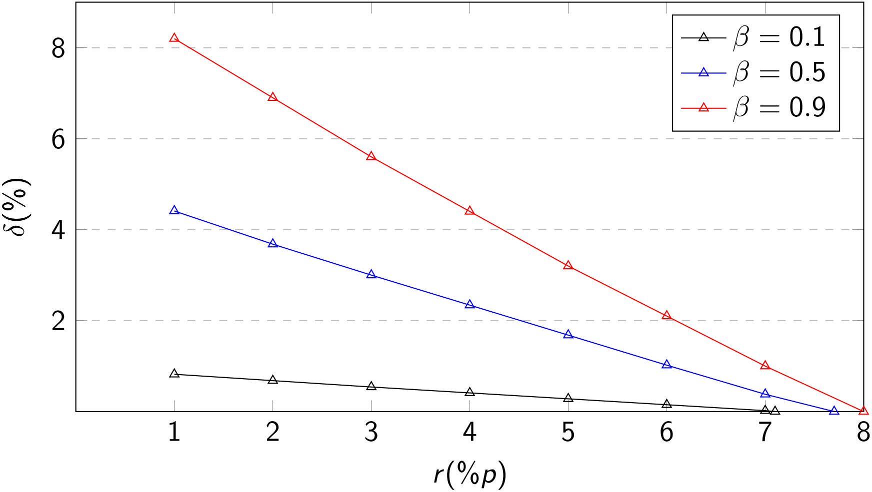

Figure: Effects of Discount by r, $p = 1.3w$, $S = 80$, U=3 weeks

Customer Acceptance as a Function of Discount

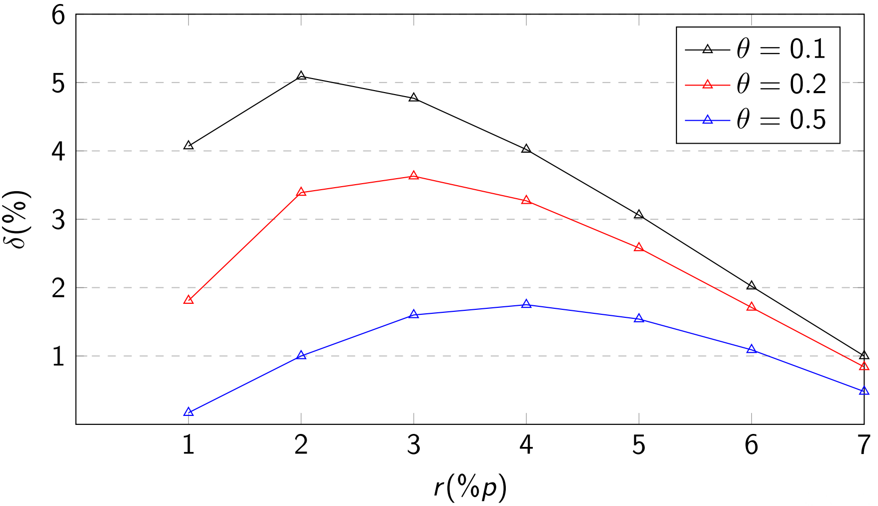

Figure: Effects of Discount by $\beta(r)$, $U = 3weeks$

Average Travelled Distance

Table: Average Travelled Distance $c_u = 0.05w, g = p - r,\ r = 0.01p$

Max Waiting Time	No Discount Case T	No Discount Case F	No Discount Case Dist	Discounted Case β	Discounted Case T	Discounted Case F	Discounted Case Dist	$\delta_{avgdist}(\%)$
3 weeks	0.29	1	2276.84	0.1	0.29	0.8	2267.81	0.396
				0.5	0.3	0.8	2196.59	3.525
				0.9	0.26	0.77	2018.86	11.330
6 weeks	0.29	1	2276.84	0.1	0.29	0.59	2255.68	0.930
				0.5	0.29	0.59	2095.03	7.985
				0.9	0.26	0.54	1720.36	24.441

Conclusion

- In this paper, a continuous review inventory - distribution problem with discount for postponing demand is presented with Net Present Value profit.
- Controlling the demand by discounts is an important strategy for increasing profit as shown in numerical experiments.
- Considering the delivery system as well makes these benefits even larger.
- We also find that that transportation distances can be significantly reduced, which will also translate into reduced CO_2 emissions to the environment.
- Further research in this area should concentrate on stochastic demand and dependent customer acceptance of discount

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